MOBILE ROBOT CONTROL ARCHITECTURE VIA CONTROL OUTPUT FUSION: STABILITY ISSUES

E. O. Freire*[†], T. F. Bastos-Filho[†], M. Sarcinelli-Filho[†], R. Carelli[‡]

* Grupo de Pesquisa em Automação do Instituto de Tecnologia e Pesquisa da Universidade Tiradentes
Av. Murilo Dantas, 300, Farolândia 49032-490 Aracaju/SE BRAZIL
```
fax: +55 79 218 2200
e-mail: eduardo_freire@unit.br
```

[†] Departamento de Engenharia Elétrica, Universidade Federal do Espírito Santo
Av. Fernando Ferrari, s/n 29060-900 Vitória/ES BRAZIL

[‡] Instituto de Automática, Universidad Nacional de San Juan
Av. San Martin Oeste, 1112 5400 San Juan Argentina

Keywords: Mobile robots; Robot control; Data fusion; Information filter; Lyapunov functions.

Abstract

A new architecture proposed by the authors in previous papers for controlling the navigation of a mobile robot, called fusion of the output of different controllers, is considered again. The novelty here included is the analysis of the stability of such control architecture. Both a formal Lyapunov-type analysis and a conjecture based on energy considerations are presented. In addition, a supervisor is included in the original control architecture in order to allow detecting changes in the robot navigation phases and ensuring the accomplishment of the stability conjecture. The control system thus modified is implemented in a commercial robot and practical experiments are run. Their results are presented in order to illustrate the system performance.

1 Introduction

An important issue in mobile robot control consists of making a decision on which action should be taken in the next time instant. This kind of problem is known as *Action Selection Problem* (ASP) or *Behaviour Co-ordination Problem* [8]. Control architectures used to solve this problem are known as A*ction Selection Mechanisms* (ASM), which can be grouped in two major categories: *arbitration schemes* and *command fusion schemes*.

Arbitration schemes are suitable for behaviour selection (a single behaviour gets the entire control of the system at each moment). They can be classified into three categories: *Priority Based*, *Winner-takes-all* and *State Based* mechanisms. Examples of arbitration schemes include the Subsumption Architecture, Discrete Event Systems and Activation Networks [8].

By its turn, command fusion schemes accepts a set of behaviours sharing the control of the whole system at each moment. Command fusion schemes can be distributed into four categories: *Voting* (e. g. DAMN [9]), *Superposition* (e. g. AuRA [1,2]), *Multiple Objective* (e. g. *Multiple Objective Decision-Making Control* [8]) and *Fuzzy Logic* (e. g. *Multivaluated Logic Approach* [10]) mechanisms. Another example of a command fusion ASM is the dynamic approach to behaviour-based robotics [3].

This work addresses a recently proposed command fusion ASM consisting of the fusion of the output of different controllers through a decentralised information filter (DIF) [4,5]. In particular, a stability analysis of the whole control system is

here developed. Both a formal Lyapunov-type analysis and a stability conjecture based on energy considerations are addressed. Besides, a supervisor is added to the original architecture to detect changes in the current robot navigation-phase. Upon detecting a meaningful change, this supervisor acts in the sense of ensuring the accomplishment of the stability conjecture.

These specific topics are addressed hereinafter in the paper. Section 2 describes how the architecture in [4,5] changes when the supervisor is included. By its turn, Section 3 presents the stability analysis, which is divided in two parts. The first one considers that the controllers included in the fusion system have the same control objective. In the second part, a conjecture regarding the stability of the fusion of the output of different controllers with different control objectives is proposed. In the sequence, Section 4 presents some experimental results to illustrate the performance of the proposed architecture and to support the statements in the

previous section. Finally, Section 5 outlines the main conclusions.

2   The Modified Control Architecture

The control architecture proposed in [4,5] is based on the fusion of the output of a set of controllers by using a decentralised information filter (DIF). Figure 1 represents an implementation of that control architecture for a robot navigating inside an office building. The novelty is the presence of the supervisory system added to the architecture, which is responsible for assuring the whole system stability, as it will be discussed in Section 3. As shown in the figure, each controller receives sensorial information and produces linear/angular velocities as its output, which are inputted to some local information filters. These local filters plus a global information filter is referred to as the decentralised information filter [4].
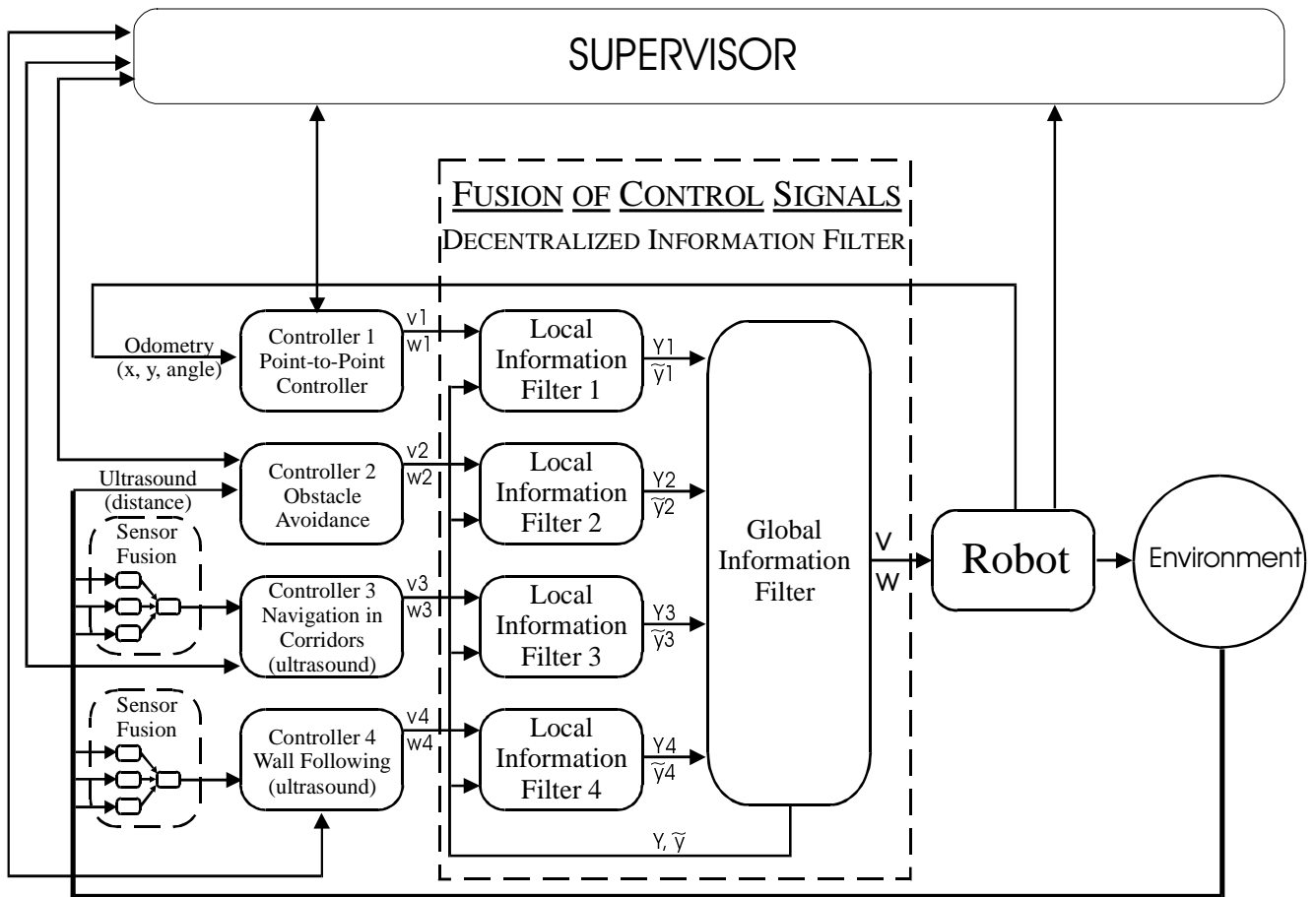

Figure 1: The proposed architecture including the supervisory system.

A covariance measuring the confidence of the observed data is associated to each local filter. The output of the global information filter is closer to the output of the local information filter associated to the lowest covariance (the more reliable output). This way, the system combines information on the angular and linear velocities coming from different controllers using the DIF, which is an optimised fusion-method [7].

When fusing the output of the controllers, the covariance represents a measure of how suitable each controller is, regarding the current environmental condition. The lower the covariance associated to a certain controller is, the more suitable it is. Thus, if a reliable inference of the environmental condition is available, a suitability degree can be associated to each controller at each instant. The environmental conditions are inferred from the information coming from the sensing system (here a set of ultrasonic sensors) or from information provided by the supervisory system.

The way the information coming from the sensing system is used to define the covariance associated to each controller is through a fuzzy logic system, as described in [4]. On the other hand, the way the supervisor acts is described in the next section.

3   Analysing the New Control System

In order to design a control system using the architecture of Figure 1, one needs to fulfil some requirements. First, the control system should be guaranteed to comply with some "good behaviour" conditions, which we will try to express as a stability condition. As part of this condition, the different controllers used should be stable in the Lyapunov sense, what ensures the assignment of energy functions to them (normalised Lyapunov functions). This allows defining an overall energy function as the sum of the energy functions associated to all the controllers included in the system. Second, the environment is considered as a partially structured one. This means that the robot has no previous information about the world model except for very general definitions as indoors plain environment. On the other hand, it should be guaranteed that a destination point is set in a free area of it.

Regarding the stability issue, we consider two navigation cases related to the proposed control structure. In the first one the active controllers in certain navigation condition are such that they have a common control objective. In the second one the more general case of different controllers having different control objectives is regarded. Both cases are analysed in the following subsections. For the first one a rigorous Lyapunov stability proposal is formulated, while for the second case a conjecture for the "good behaviour" of the control system is proposed, with basis on energy functions. In order to guarantee the accomplishment of this conjecture a supervisor is designed as part of the whole architecture.

3.1 Controllers Having a Common Control Objective

In this subsection, the stability of the control system resulting from the fusion of different controllers with the same control objective is analysed. For example, suppose that three controllers are available to accomplish the task of navigating along a corridor. The first controller is based on information provided by an ultrasonic system that informs to the control system the relative position of the robot related to the middle of the corridor. The second controller tries to equalise the optic flow measured on the right and on the left corridor walls. The last controller equalises the angle formed by the junction of the walls and the floor on the image plane. Each one of these controllers generates an angular velocity control signal.

As a first step, consider that only one controller is used, as it is depicted in Figure 2. Considering that the robot angular velocity dynamics can be modelled as
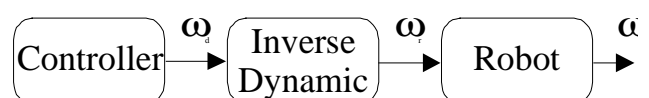

Figure 2: System with one controller.

$$\frac{\omega}{\omega_r} = \frac{k}{s^2 + as + b} \qquad (1)$$

so that ω_r can be written as

$$\omega_r = \frac{1}{k}\left(\ddot{\omega} + a\dot{\omega} + b\omega\right) \qquad (2)$$

Using an inverse dynamics control law given by

$$\omega_r = \frac{1}{k}\left(\eta + a\dot{\omega} + b\omega\right) \qquad (3)$$

where

$$\tilde{\omega} = \omega_d - \omega \qquad (4)$$

$$\eta = \ddot{\omega}_d + k_d\dot{\tilde{\omega}} + k_p\tilde{\omega} \quad k_p, k_d > 0 \qquad (5)$$

the closed loop equation for exact knowledge of the robot dynamics is given by

$$\eta = \ddot{\omega} \qquad (6)$$

Then, replacing the control law of Equation (5) one gets

$$\ddot{\tilde{\omega}} + k_d\dot{\tilde{\omega}} + k_p\tilde{\omega} = 0 \qquad (7)$$

which implies that $\tilde{\omega}(t) \to 0$.

Now, if more than one controller with the same control objective is used – like in Figure 3 – and supposing that all the state variables associated to them are available at each time instant, one can write the set of equations

$$\omega_{r1} = \frac{1}{k}\left(\eta_1 + a\dot{\omega} + b\omega\right)$$

$$\omega_{r2} = \frac{1}{k}\left(\eta_2 + a\dot{\omega} + b\omega\right)$$

$$\vdots$$

$$\omega_{rn} = \frac{1}{k}\left(\eta_n + a\dot{\omega} + b\omega\right)$$
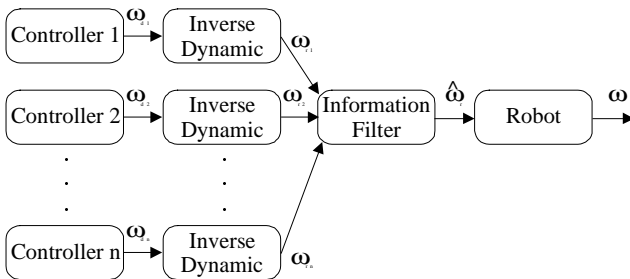
Then, the fused control signal is

Figure 3: Output fusion of different controllers.

$$\hat{\omega}_r = \frac{1}{k}\left(\hat{\eta} + a\dot{\omega} + b\omega\right) \qquad (8)$$

For an ideal control command $\omega_d = \omega_{di} + \Delta\omega_{di}$ it corresponds an ideal η such that

$$\eta = \eta_1 + \Delta\eta_1$$

$$\eta = \eta_2 + \Delta\eta_2$$

$$\vdots$$

$$\eta = \eta_n + \Delta\eta_{n1}$$

what results in

$$\eta = \hat{\eta} + \Delta\hat{\eta} \qquad (9)$$

By equating Equations (8) and (2) one gets

$$\hat{\eta} = \ddot{\omega} \qquad (10)$$

and finally

$$\hat{\eta} = \eta - \Delta\hat{\eta} = \ddot{\omega} \qquad (11)$$

taking Equation (9) in account.

Now, from Equations (5) and (11) it is possible to write the following dynamics for the angular velocity error

$$\ddot{\tilde{\omega}} + k_d\dot{\tilde{\omega}} + k_p\tilde{\omega} = \Delta\hat{\eta} \qquad (12)$$

Defining the state vector $x = \begin{bmatrix} \tilde{\omega} & \dot{\tilde{\omega}} \end{bmatrix}^T$, Equation (12) can be written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \boldsymbol{\delta}(\mathbf{x}) \qquad (13)$$

where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -k_p & -k_d \end{pmatrix} \quad \boldsymbol{\delta}(\mathbf{x}) = \begin{pmatrix} 0 \\ \Delta\hat{\eta} \end{pmatrix}$$

Now, it is easy to prove that the system described by Equation (13) has an ultimately bounded solution [6]. This means that there are b, $c>0$ such as for each $\alpha \in (0,c)$ there is a positive constant $T = T(\alpha)$ such that

$$\|\mathbf{x}(t_0)\| < \alpha \Rightarrow \|\mathbf{x}(t)\| \le b \ \forall t \ge t_0 + T(\alpha) \qquad (14)$$

where b is the ultimate bound.

Taking the following Lyapunov candidate

$$V = \mathbf{x}^T\mathbf{P}\mathbf{x}, \quad \mathbf{P} = \mathbf{P}^T > 0 \qquad (15)$$

its time derivative is

$$\dot{V} = -\mathbf{x}^T\mathbf{Q}\mathbf{x} + 2\mathbf{x}^T\mathbf{P}\boldsymbol{\delta}(\mathbf{x}) \qquad (16)$$

where

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q} \qquad (17)$$

Taking bounds on both terms of Equation (16)

$$-\mathbf{x}^T\mathbf{Q}\mathbf{x} \leq -\lambda_{\min}(\mathbf{Q})\|\mathbf{x}\|^2 \qquad (18)$$

$$\left\|2\mathbf{x}^T\mathbf{P}\right\| \leq 2\lambda_{\max}(\mathbf{P})\|\mathbf{x}\| \qquad (19)$$

one can write

$$\dot{V} \leq -\lambda_{\min}(\mathbf{Q})\|\mathbf{x}\|^2 + 2\lambda_{\max}(\mathbf{P})\|\mathbf{x}\|\|\boldsymbol{\delta}(\mathbf{x})\| \qquad (20)$$

From Equation (13)

$$\|\boldsymbol{\delta}(\mathbf{x})\| \leq |\Delta\hat{\eta}| \qquad (21)$$

Regarding Equation (20) one can write

$$\dot{V} \leq -(1-\theta)\lambda_{\min}(\mathbf{Q})\|\mathbf{x}\|^2 - \theta\lambda_{\min}(\mathbf{Q})\|\mathbf{x}\|^2 + \\ + 2\lambda_{\max}(\mathbf{P})\|\mathbf{x}\||\Delta\hat{\eta}| \qquad (22)$$

with $0<\theta<1$. Finally, it results

$$\dot{V} \leq -(1-\theta)\lambda_{min}(\mathbf{Q})\|\mathbf{x}\|^2, \forall\|\mathbf{x}\| \geq \frac{2\lambda_{max}(\mathbf{P})|\Delta\hat{\eta}|}{\lambda_{min}(\mathbf{Q})\theta} \quad (23)$$

so that the ultimate bound [6] is

$$b = \frac{2\lambda_{max}(\mathbf{P})}{\lambda_{min}(\mathbf{Q})}\sqrt{\frac{\lambda_{max}(\mathbf{P})}{\lambda_{min}(\mathbf{P})}}\frac{|\Delta\hat{\eta}|}{\theta} \qquad (24)$$

As a Kalman-type filter is being used to fuse the control signals, the ultimate bound on the standard deviation of ultimate error is smaller than that corresponding to the errors produced by each controller.

The more general case of the output fusion of different controllers with different control objectives is addressed in next subsection.

3.2 Controllers with Different Control Objectives

When the controllers involved in the fusion process do not have the same control objectives, the stability analysis made in the previous subsection is not valid anymore. An example is the system presented in Figure 1, for which the four controllers have different control objectives (goal seeking, obstacle avoidance, wall following and corridor navigation). In this case, a conjecture based on navigation phases and energy associated to each controller is proposed, which is now discussed.

When navigating from an initial point to a destination point (goal seeking) the robot goes through several navigation phases. A navigation phase is a part of the path followed by the robot where just one control objective dominates. If the main control objective changes, a navigation phase is over and another one starts. The control system detects a change in the navigation phase when the energy function assigned to at least one of the controllers grows faster then it would grow normally (due to the limited linear and angular velocities). This kind of growth will be called an abrupt one, while a normal growth will be called a gradual one. Examples of navigation phases are wall following, obstacle avoidance, corridor following, goal seeking, etc.

Thus, an important detail when designing a control system using the architecture in Figure 1 is that at least one controller corresponding to each distinct navigation phase the robot will face should be provided.

Now, regarding the stability of the controllers used, the overall system energy is supposed to decrease while the robot remains in the same navigation phase. In order to ensure this, a *supervisory system* is included in the control architecture to monitor the energy function of each controller and the energy function of the entire system. Then, if the energy function of the system starts gradually growing, the controllers whose energy functions are gradually growing are eliminated of the fusion process. Notice that this is equivalent to make the covariance associated to them infinite.

As the environment is unknown, the kind and the number of navigation phases the robot should pass through to accomplish its task are also unknown. It is also impossible to know the exact time at which a navigation phase change will occur. Because of this, one can consider the transition between two navigation phases as a perturbation. For this reason, the system energy function is allowed to grow during the transition between two subsequent navigation phases.

The supervisor must also eliminate of the fusion process the controllers that are out of context. A specific controller is out of context when its state

variables are not available. An example of this situation is a robot in the middle of a very big room. As its sensing system (only ultrasonic sensors, in this paper) does not detect any wall, the wall following controller and the corridor following controller can not operate once the robot does not detect a wall or a corridor to follow.

When the controllers used in the fusion process have different control objectives, a formal demonstration of the overall system stability is not possible. However, the system requirements here presented ensure that the energy function of the system decreases during a navigation phase. On the other hand, it is allowed to grow in the transition between two navigation phases, once this can be viewed as a perturbation. This is accomplished by the presence of the *supervisor* included in the system (Figure 1).

To validate this conjecture, several experiments were executed and some of them are presented in next section.

4 Experimental Results

In order to evaluate how the modified control architecture performs and to check the accomplishment of the stability conjecture proposed in Subsection 3.2, four practical experiments consisting in guiding a robot navigating inside an office building are considered.

The experiments were run using a PIONEER 2DX mobile robot having sixteen ultrasonic sensors (only ten are effectively used) and a single CCD camera (not used here). The driven wheels and ultrasonic sensors are controlled by a Siemens 20 MHz 88C166 micro-controller. The navigation is controlled from onboard computer (a 500 MHz K6-II PC) running the control architecture of Figure 1.

To evaluate the performance of the control system during each experiment, four indexes have been considered [8]. Tables 1, 2, 3 and 4 show the resulting values for the performance indexes of the four experiments, respectively, including the ideal values for such indexes. The safety index indicates the minimal distance measured by the ultrasonic sensors along the robot path, thus indicating the

risk of collision. As shown, the robot navigation during the experiment was quite safe. The average velocity (linear velocity) index indicates the average linear velocity along the robot path. As one can see, the fusion of distinct control signals makes the robot to navigate a little slower. Finally, the smoothness index is measured calculating the average value of the magnitude of the difference between the current and the previous robot orientation, thus showing how smoothly the manoeuvres are performed. As one can see, the
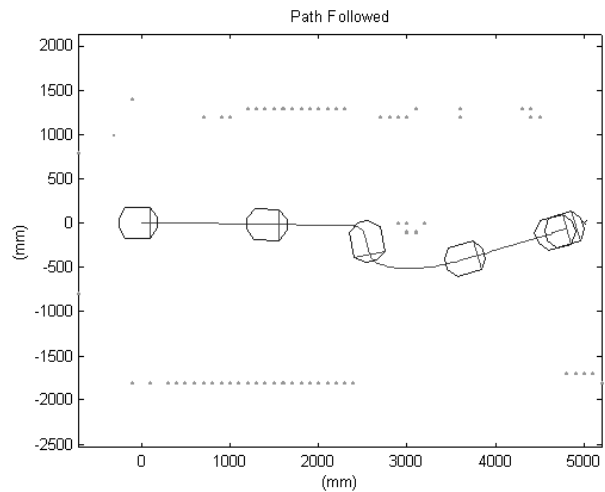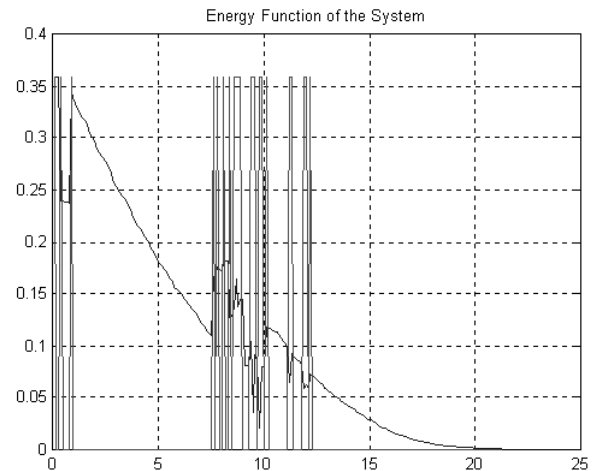

Figure 4: Path followed in Experiment #1.

Figure 5: Energy function for Experiment #1.

Index	Obtained Value	Ideal Value
Safety	252 mm	1250 mm
Average Velocity	242 mm/s	300 mm/s
Smoothness	0.90^o	0°
Travelled Distance	5.15 m	5.00 m
Elapsed Time	21.30 s	16.67 s

Table 1: Performance evaluation indexes.

proposed architecture effectively allows very smooth manoeuvres.

The first experiment consists of avoiding an obstacle located in the robot path. Figure 4 shows the path followed by the robot, while Figure 5 shows how the system energy function behaves. In this figure, an additional line is plotted to represent the transition between two navigation phases: it is different from zero during a transition between navigation phases and zero otherwise.

The second experiment consists in avoiding a V-form obstacle. It demonstrates the capability of the control architecture to avoid local minima. Figure 6 shows the path followed by the robot, while Figure 7 shows how the system energy function behaves.

The third experiment consists in guiding the robot
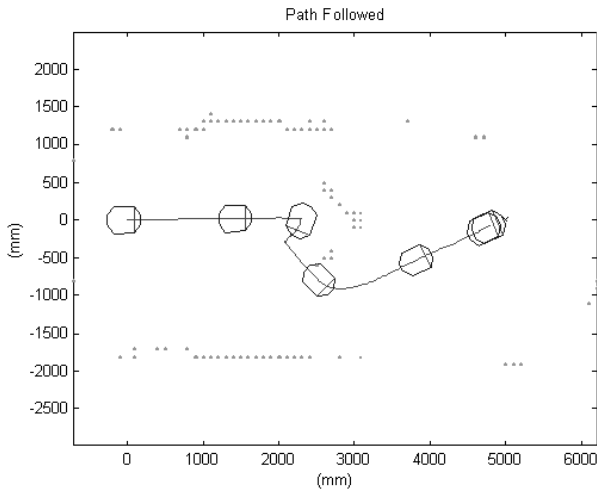
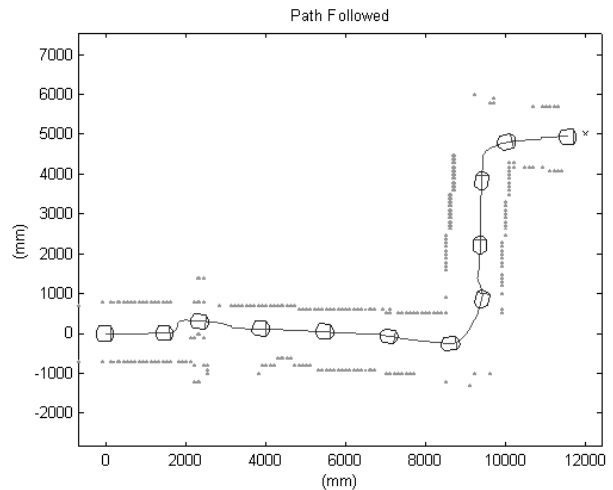Figure 6: Path followed in Experiment #2.
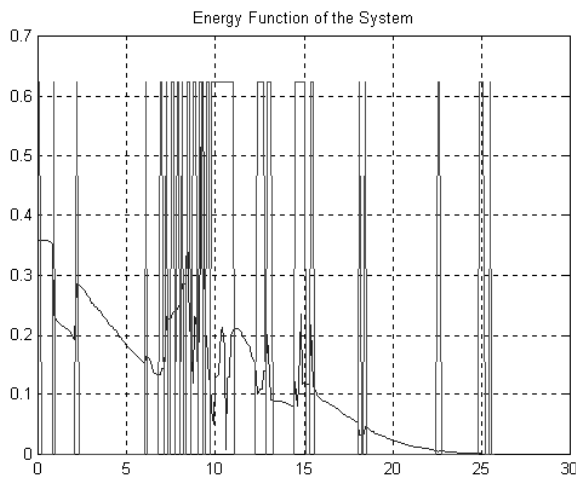

Figure 8: Path followed in Experiment #3.
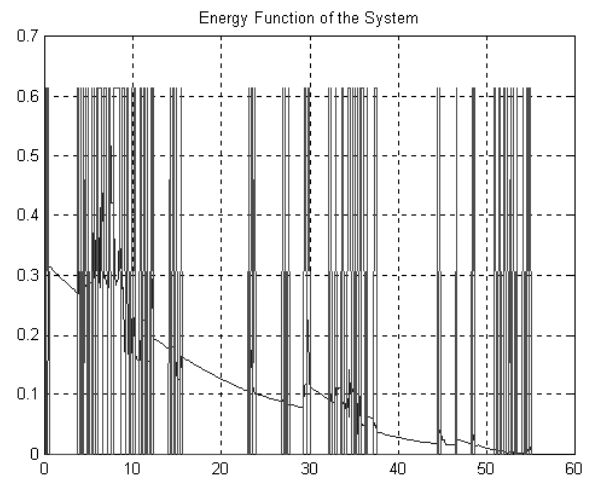

Figure 7: Energy function for Experiment #2.

Figure 9: Energy function for Experiment #3.

Index	Obtained Value	Ideal Value
Safety	170 mm	1250 mm
Average Velocity	229 mm/s	300 mm/s
Smoothness	1.27^o	0°
Travelled Distance	5.91 m	5.00 m
Elapsed Time	25.80 s	16.67 s

Table 2: Performance evaluation indexes.

Index	Obtained Value	Ideal Value
Safety	172 mm	500 mm
Average Velocity	293 mm/s	300 mm/s
Smoothness	0.89^o	0.32^o
Travelled Distance	16.14 m	17.00 m
Elapsed Time	55.10 s	56.67 s

Table 3: Performance evaluation indexes.

from an initial point (at the co-ordinates [0m, 0m]) to a destination point (at the co-ordinates [12m, 5m]) in an office building. While seeking its final goal, the robot should navigate along corridors while avoiding obstacles in its path. Figure 8 shows the path followed by the robot, while Figure 9 shows how the system energy function behaves.

Finally, the fourth experiment consists in guiding the robot from an initial point (at the co-ordinates [0m, 0m]) to a destination point (at the co-ordinates
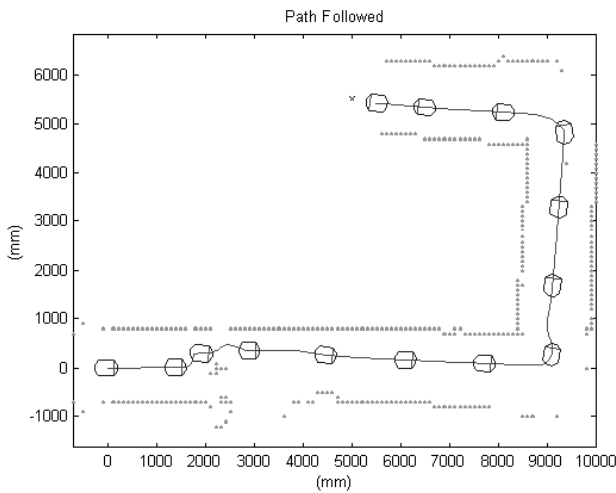

Figure 10: Path followed in Experiment #4.

Figure 11: Energy function for Experiment #4.

Index	Obtained Value	Ideal Value
Safety	142 mm	500 mm
Average Velocity	287 mm/s	300 mm/s
Smoothness	0.99^o	0.31^o
Travelled Distance	18.11 m	17.50 m
Elapsed Time	63.20 s	58.33 s

Table 4: Performance evaluation indexes.

[5m, 5.5m]) in the same office building. This experiment also demonstrates the capability of the control architecture to avoid local minima. Figure 10 shows the path followed by the robot, while Figure 11 shows how the system energy function behaves.

5 Conclusions

Stability aspects associated to specific control architecture recently proposed by the authors are here discussed. The basis of such architecture is the fusion of the output of several different stable controllers, some of them having the same control objective and others having distinct control objectives. A version of this architecture including one controller responsible for goal seeking, one controller responsible for obstacle avoidance, one controller responsible for wall following and one controller responsible for navigating in a corridor is considered as example.

Actually, it is formally demonstrated that the fusion of the output of different controllers having the same control objective has an ultimately bounded solution. In addition, the control signal resulting of the fusion is better than the output of each single controller in the sense that the variance of the ultimate error is smaller.

The stability of the output fusion of different controllers with different control objectives is also addressed. In this case, a stability conjecture is presented which is validated through several experiments, from which four are presented.

The conclusion based on the mathematical analysis and the experimental results here presented is that the fusion of the output of different controllers effectively presents a "good behaviour". This means that the robot does not loose its final objective either when obstacles are present in its path or when its working environment determines a temporary deviation of the final goal.

Acknowledgements

References

[1] R. C. Arkin and T. Balch. "AuRA: principles and practice in review", *Experimental and Theoretical Artificial Intelligence*, **9**, pp. 175-189, (1997).

[2] R. C. Arkin. *Behaviour-based robotics*. The MIT Press, Reading, MA, USA, (1998).

[3] E. Bicho. *The dynamic approach to behaviour-based robotics*. PhD. Thesis, University of Minho, Portugal (1999).

[4] E. O. Freire, T. F. Bastos-Filho, M. Sarcinelli-Filho, R. Carelli, R. and O. Nasisi. "A new mobile robot control architecture via control output fusion", To appear in the *Proceedings of the XV IFAC World Congress*, Barcelona, Spain, (2002).

[5] E. O. Freire, R. Carelli, V. Mut, C. M. Soria, T. F. Bastos-Filho and M. Sarcinelli-Filho. "Mobile robot navigation based on the fusion of control signals from different controllers" *Proceedings of the 2001 European Control Conference*, pp. 1828-1833. Porto, Portugal, (2001).

[6] H. K. Khalil. *Non-linear systems*, Second Edition, Prentice-Hall, USA, (1996).

[7] A. G. O. Mutambara. *Decentralised estimation and control for multi-sensor systems*. CRC Press, USA, (1998).

[8] P. Pirjanian. "Multiple objective behaviour-based control", *Robotics and Autonomous Systems*, **31**, pp. 53-60, (2000).

[9] J. Rosenblatt. *DAMN: A distributed architecture for mobile navigation*. PhD Thesis, Carnegie Melon University, USA, (1997).

[10] A. Saffiotti, K. Konolige and E. Ruspini. " A multivaluated logic approach to integrating planning and control", *Artificial Intelligence*, **76**, pp. 481-526, (1995).